Implementation of Fuzzy Logic Selected Applications

Texas Instruments

Contents

Introduction

The name "Fuzzy Logic" seems to imply an imprecise methodology that is useful only when accuracy is not necessary or important. That is what many people assume when they first hear about fuzzy logic—and understandably so. In a world increasingly manipulated by computers with their absolute "1" or "0" and "on" or "off" concepts, a term like fuzzy logic suggests inaccuracy or imprecision. Even Webster's dictionary defines "fuzzy" as:

fuzz·y (–ē) adj. 2. not clear, distinct, or precise; blurred

This is not true of fuzzy logic. Fuzzy logic can address complex control problems, such as robotic arm movement, chemical or manufacturing process control, antiskid braking systems, or automobile transmission control with more precision and accuracy, in many cases, than traditional control techniques have.

Fuzzy logic was invented and named by Lotfi Zadeh, a professor at the University of California at Berkeley. Fuzzy logic is a methodology for expressing operational laws of a system in linguistic terms instead of mathematical equations. Many systems are too complex to model accurately, even with complex mathematical equations, but fuzzy logic's linguistic terms provide a useful method for defining the operational characteristics of such a system. These linguistic terms are most often expressed in the form of logical implications, such as If – Then rules:

If air_temp is WARM, then set fan_speed to MEDIUM.

The terms WARM and MEDIUM are actually sets that define ranges of values known as membership functions. By choosing a range of values instead of a single discrete value to define the input variable "air_temp", you can control the output variable "fan_speed" more precisely. Fuzzy logic controllers can often improve the performance of a control system by reducing the chance of wild functions in the output that may be caused by variations in the measured input variables.

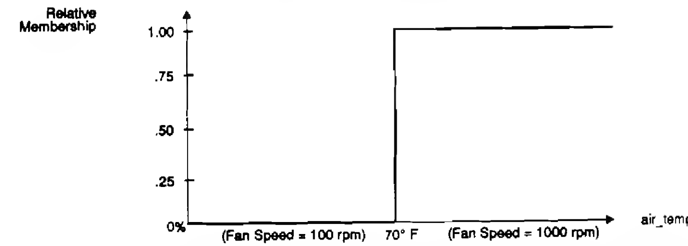
The Traditional Approach

To illustrate the difference between fuzzy logic and the traditional approach, here is a control problem. First, consider how the traditional—often called "crisp"—controller would handle it:

If air_temp is ≥ 70° Fahrenheit, then set fan_speed to "1000 rpm".
If air_temp is < 70° Fahrenheit, then set fan_speed to "100 rpm".

A nonfuzzy, or "crisp," controller relies on a discrete valued decision point. For this type of system, the input must reach an exact value before the control system reacts in a certain way. Even small variances in this input value may cause the output to react drastically differently. For instance, if the temperature is 70° or above, the first rule will set the fan_speed to 1000 rpm. If the temperature is below 70°, the second rule will set the fan_speed much lower, to 100 rpm. Figure 1 shows a diagram of this crisp valued controller.

Figure 1. Crisp Controller



What would happen if the temperature were 69.5°? Or, more importantly, what would happen to the control system if the temperature were transitioning from below 70° to above 70°? The temperature might even fluctuate back and forth slightly above or below 70° (e.g., 69.0° to 71.0°). This would cause the control system to alter the fan speed wildly for changes in the input variable air_temp, although the temperature change may not be significant.

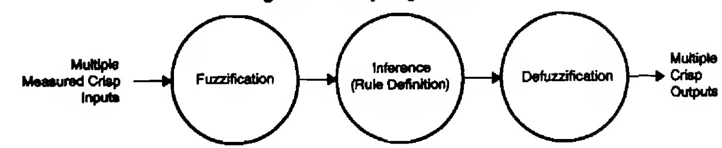
These transition points are difficult for "crisp" control systems to handle, but they are exactly where "fuzzy logic" excels.

Fuzzy Control

Fuzzy logic is implemented in three phases (see Figure 2):

1. Fuzzification (crisp input to fuzzy set mapping).
2. Inference (fuzzy rule generation).
3. Defuzzification (fuzzy to crisp output transformation).

Figure 2. Fuzzy Logic Phases

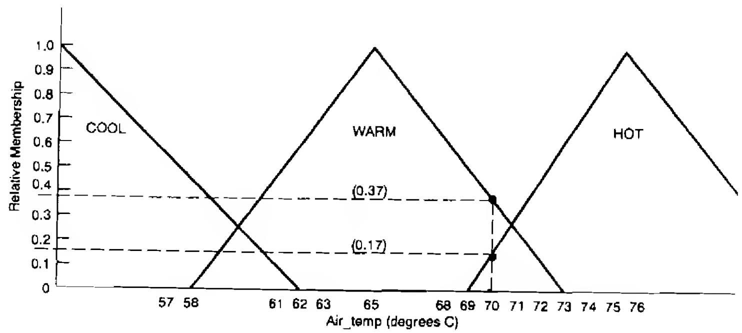


Fuzzification

In the first fuzzy logic phase—fuzzification—actual measured input values are mapped into fuzzy membership functions. As an example, a climate-control system has been developed with fuzzy logic.

To create a climate control system, we first developed membership functions for the input variable "air_temp". These membership functions are defined by both a range of values and a degree of membership. In fuzzy logic, it is important to distinguish not only which membership functions a variable belongs to, but also the relative degree to which it is a member. This gives the variable a "weighted" membership in a membership function. A variable can have a weighted membership in several membership functions at the same time. The membership functions for "air_temp" are shown in Figure 3.

Figure 3. Air_Temp Input Variable Membership Functions



As shown in Figure 3, fuzzy membership functions span a range of values and can actually overlap. Three sets of membership values are defined above for the variable "air_temp". They are COOL, WARM, and HOT. The degree of membership is found by finding the intersection point of a distinct input value on the horizontal axis with the line defining one or more fuzzy membership functions. This intersection point is assigned a corresponding value on the vertical axis to define the relative membership in a set for an actual measured input value. Notice that when "air_temp" is at a particular value, it may be contained in one or more fuzzy sets. For instance, at 70°, "air_temp" is a member of the function HOT with a relative membership of 0.17. It is also a member of the function WARM with a relative membership of 0.37. Unlike a crisp system in which a value either is or is not a member of a function, a fuzzy logic system can take action based not only on membership in a fuzzy set, but also on the degree to which a variable is included in a membership function. In this case, because "air_temp" at 70° is more WARM (0.37) than it is HOT (0.17), the controller will take that into account when defining what output action to take.

Inference Rule Definition

Once membership functions have been defined for input and output variables, a control rule base can be developed to relate the output actions of the controller to the observed inputs. This phase is known as the inference, or rule definition portion, of fuzzy logic. Any number of rules can be created to define the actions of the fuzzy controller. Some examples are shown below.

Fuzzy Logic Rule Definition

If air_temp is COOL, then set fan_speed to SLOW.
If air_temp is HOT, then set fan_speed to FAST.
If air_temp is WARM, then set fan_speed to MEDIUM.

These If-Then rules can relate multiple input and output variables. Because the rules are based on word descriptions instead of mathematical definitions, any relationship that can be described with linguistic terms can typically be defined by a fuzzy logic controller. This means that even nonlinear systems can be described and easily controlled with a fuzzy logic controller. In addition, since variables have weighted memberships—in particular membership functions—the rules that are composed of these variables are weighted as well. This means that different rules have different impacts on the controller, according to the measured input variable. For a multiple-input/multiple-output system with many defining rules, a wild fluctuation in any single input will be tempered by these rule weightings. Because of this, fuzzy logic systems are very robust and often allow many rules to be removed or altered without significantly impacting the controller.

Defuzzification

After the fuzzy logic controller evaluates inputs and applies them to the rule base, it must generate a usable output to the system it is controlling. This may mean setting a voltage or current to a particular value to control the speed of a fan in the example above, or it may mean defining the optimal speed of a robotic arm as it nears its target. The fuzzy logic controller must convert its internal fuzzy output variables into crisp values that can actually be used by the controlled system. You can perform this portion of the fuzzy control algorithm, known as defuzzification, in several ways. Two of the most common methods are:

- maximum defuzzification method (page 6).
- centroid calculation defuzzification method (page 7).

Remember from fuzzification that in mapping input variables to membership functions, a particular measured value of the input variable determined the relative membership of that input variable in an input membership function. To determine the mapping of output variables to their corresponding output membership functions, the weighted input membership function and corresponding rule base determine the relative membership in the output function. Whatever relative membership was given to the input variable will also be given to the output variable, as assigned by its corresponding rule. For air_temp = 70°, the output variables are assigned a value that corresponds to the input value shown in Table 1.

Table 1. Fan_Speed (Membership Function Relative Membership)

Input Variable	Defining Rules	Output Variable
air_temp (WARM) = 0.37	If air_temp = WARM, then set fan_speed to MEDIUM	fan_speed (MED) = 0.37
air_temp (HOT) = 0.17	If air_temp = HOT, then set fan_speed to FAST	fan_speed (FAST) = 0.17

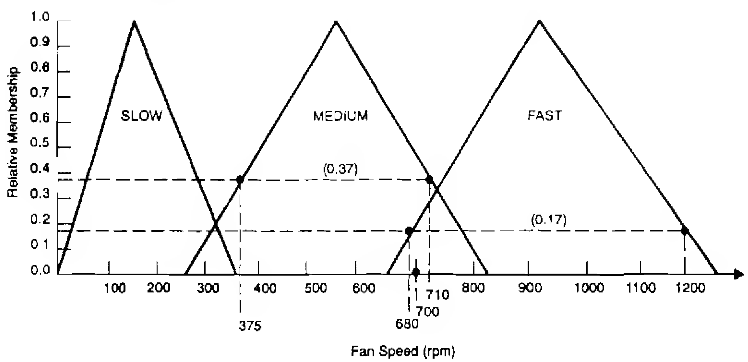
The output variable fan_speed is given the same relative mapping as the input variable air_temp that is defined by a particular rule.

Figure 4 illustrates the output variable membership functions. It this case, a distinct value on the horizontal axis is defined by the relative membership on the vertical axis. To create the actual crisp output value for the controller system output, membership functions are used with the output variable. In Table 1, the input value air_temp = 70 resulted in two weightings for fan_speed:

- Fan_speed = 0.37 was assigned to the output membership function MEDIUM.
- Fan_speed = 0.17 was assigned to the output membership function FAST.

As shown in Figure 4, the actual output value is determined by beginning at the weighting factor on the vertical axis and moving horizontally until an intersection point is reached on the lines defining its associated membership function. This intersection point is then transposed to the horizontal axis to determine the crisp output value.

Figure 4. Fan_Speed Output Variable Membership Functions



Maximum Defuzzification Method

One method of defuzzification is known as the maximum method. In this method, if more than one rule is active, the maximum relative membership is used to determine the output value. In the above example, making air_temp = 70° created two possible values for fan_speed:

fan_speed = 0.37
fan_speed = 0.17

The maximum defuzzification method provides a single output by choosing the active rule with the greatest relative membership value in the output membership function. In the preceding example, the following rule is chosen because it has the highest membership value for fan_speed.

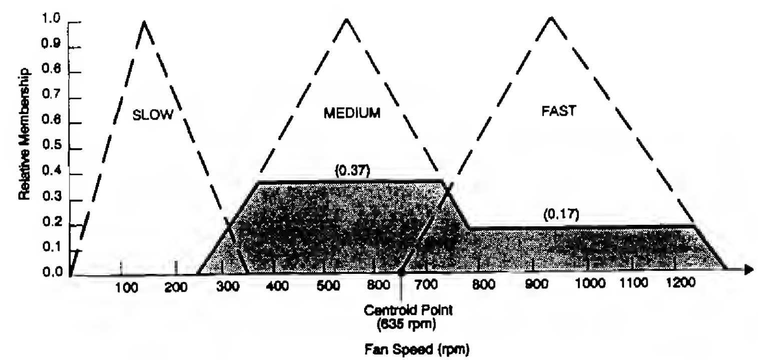
If air_temp = WARM, then set fan_speed to MEDIUM. Fan_speed (MED) = 0.37

The value 0.37 on the vertical axis intersects the membership function MEDIUM at two points—one on the positive slope (at 375 rpm) and one on the negative slope (at 710 rpm) of the function. The two points represent two possible solutions that must be resolved.

Centroid Calculation Defuzzification Method

Another method for calculating the output value is the centroid method. In this method, a weighted average of all the active rules determines an output by summing all of the applicable output variables over their relative membership values. Although this method is more computationally intensive, it creates a distinct output value based on the relative memberships of all of the active rules that apply (see Figure 5). This method eliminates the problem of multiple solutions observed with the maximum method. A processor architecture with a hardware multiply-accumulate feature like that of the TMS320 DSP family excels at this method.

Figure 5. Fan_Speed Output Centroid Calculation



Summary

By using fuzzy logic, you can simplify complex control problems that once required a high-powered microprocessor to execute in real time; you can now execute them on a low-cost Texas Instruments TMS320 DSP or TMS370 microprocessor. The following application note shows the benefits of controlling a simple DC motor with fuzzy logic using a TMS320C14 digital signal processor.

Implementation of Fuzzy Logic Servo Motor Control on a Programmable Texas Instruments TMS320C14 DSP

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Abstract

This paper describes the implementation of a fuzzy logic compensator on a Texas Instruments TMS320C14 DSP-based servo motor control development system. The system contains a real motor that is controlled by the programmable DSP. An on-chip debugger and servo motor program allowed both simple code modification and interactive control of the motor. A fuzzy logic algorithm was directly substituted for the original PID algorithm; this resulted in comparable motor response and algorithm performance. This implementation proves the feasibility of real-time fuzzy logic-based servo motor control on a real system.

Introduction

Fuzzy logic is relatively new theory. Most of the readily available hands-on fuzzy logic system examples have been software simulations or bulky real systems. However, a TI commercial microprocessor (the TMS320C14) can serve as a simple, real-time, real-system platform for applying and investigating fuzzy logic. This facilitates both the understanding and implementation of fuzzy logic as a real-time programmable solution for the general engineering public.

Servo motor control is a viable and useful implementation. A programmable PID motor control board uses a Texas Instruments TMS320C14 chip and has an actual motor whose performance can be observed. Faster and newer parts are available, but the 'C14 is optimized for motor control with such features as on-board pulse-width modulation (PWM) generation capabilities. The board also has on-chip debugger code and interactive PID control code. The PID compensator code (written in TMS320 assembly language, which is upward-compatible with code executed by such newer fixed-point TI DSPs as the TMS320C25 or TMS320C50) uses position and velocity of the motor for inputs and motor input current as the output. The control code will allow the operator to interactively change such values as servo position and velocity and to monitor position error. Modifying the code for fuzzy logic required replacing the PID compensator section of the code with a fuzzy logic compensator.

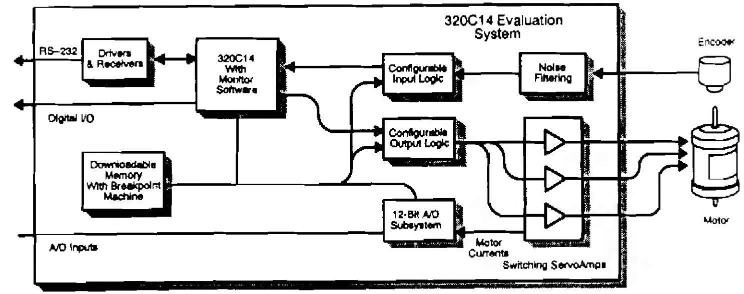
The membership function for the compensator defines the error between present motor position and the desired (command) position of the controller. Five linguistics variables characterize the function: negative medium, negative small, zero, positive small, and positive medium. The function is represented as overlapping isosceles right triangles for ease of fuzzification. Various rules for an inverted pendulum control system (ball and stick) are explained in [1]. These same eleven rules were used for servo motor control.

The algorithm's three sections are implemented as a series of software loops: fuzzification (i.e., input evaluation), fuzzy inference (rule contribution that uses a table look-up), and defuzzification (using center-of-gravity method). It is based on an algorithm developed for [4]. Each section uses various arrays that are modified in that section. The sizes of these loops are directly proportional to the number of inputs, outputs, and rules used in the system.

Servo Motor System (Power-14)

The heart of the Power-14 board is a Texas Instruments TMS320P14 chip (one-time programmable TMS320C14). (See Figure 1).

Figure 1. Power 14 System



You communicate to the board through an RS-232 serial port connection by using standard terminal or terminal emulation software (such as Procomm). The 'P14 peripherals are optimized for control applications. An event manager can be operated in a PWM (pulse-width modulation) mode that is ideal for motor control. Monitor code is loaded into 'P14 external memory and run, which provides a command line debugger. The debugger has all standard debugger functions, such as memory dumps and modification, stepping through code, breakpoints, etc. The monitor can be used to load and run the servo motor program with PID compensator. The program is interactive and lets you control the motor from the keyboard. Position, velocity, and the PID values can be set. Data acquisition functions allow an ASCII text input stimulus table to be loaded onto the Power-14, an acquisition run to be executed, and the resulting ASCII output table to be written to a PC file for graphing. The rest of the board contains support circuitry: amplifiers for the motor and a serial port interface. An encoder on the motor is used as a position sensor for a compensator input. The velocity is found by executing a back-difference. Note that there is no separate velocity sensor.

PID Implementation

The source code for this system implements the PID compensator in one file (See Appendix A). The algorithm is a direct implementation of the PID equation. The Proportional, Integral, and Differential variables are derived from the motor encoder sensor detecting position. On each cycle, the present position is taken from the encoder and stored in Position. The error is found by subtracting Position from DesiredPosition and storing it in ErrNow. Thus, ErrNow is the position input for the proportional section of the PID. A back-difference is then taken with ErrNow and ErrLast (the error in the previous cycle) to give ErrDiff. ErrDiff is an approximation of the velocity and is therefore the second input (Differential) of the PID. The Integral is found by adding the ErrNow value and storing it in Kintegrator. The following equation then holds the compensator output:

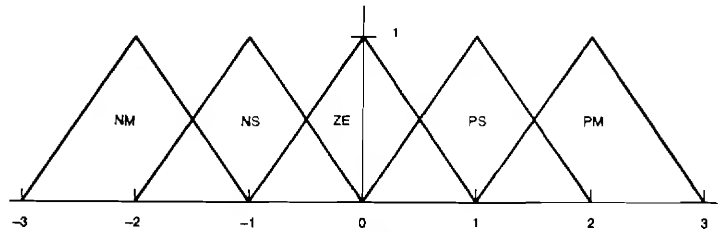
$$PWM = 9 * P * ErrNow + 9 * I * Kintegrator + 9 * D * ErrDiff \qquad (1)$$

The value is scaled for the PWM mode and stored in NewServo. Thus, for the compensator output, the actual motor current input value is converted to a PWM value for implementation. The PWM output is then sent to power amplifiers that finally drive the motor. The PWM frequency can be controlled from the PID program.

Fuzzy Logic Theory for Servo Motor Control

The membership function for this system is simple. Five fuzzy logic ranges (linguistic variables) were chosen to remain consistent with [1] and used for both inputs. Sets of values are represented as overlapping isosceles right triangles (Figure 2).

Figure 2. Membership Function



Key
PM=positive medium
PS=positive small
Z=zero
NS=negative small
NM=negative medium

Two input variables, position (Theta) and velocity (dTheta) of the motor, are used in this fuzzy logic system. One output variable, motor current, is used, which will be proportional to the PWM output that is actually written. The rules for the compensator as mentioned were taken from [1]. Thus Theta, dTheta, and motor current operate according to the following "if a and b, then c" rules, as shown in Table 1.

Table 1. List of Rules

If Theta =	And dTheta =	Then Motor Current =
Z	Z	Z
PS	Z	NS
PM	2	NM
NS	Z	PS
NM	Z	PM
Z	NS	PS
Z	NM	PM
Z	PS	NS
2	PM	NM
PS	NS	Z
NS	PS	Z

These rules can then be indexed according to the scale shown in Figure 2. The mapping seen in Table 2 will be used in TMS320 programming.

Table 2. Indexed List of Rules

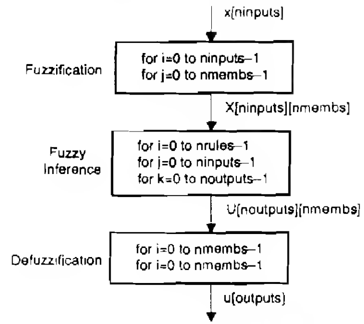
If Theta =	And dTheta =	Then Motor Current =
0	0	0
1	0	–1
2	0	–2
–1	0	1
–2	0	2
0	–1	1
0	–2	2
0	1	–1
0	1	–2
1	–1	0
–1	1	0

The defuzzification is done by the center-of-gravity method.

Fuzzy Logic Implementation

Arrays are used in the fuzzy logic calculations and modified in the various loops that implement the compensator. Appendix B lists the TMS320C14 code. Figure 3 shows the three sections of the compensator: fuzzification, fuzzy inference, and defuzzification. Notation for the arrays follows C language standard, with subscripts from 0 to n–1. The 'C14 algorithm is based on an algorithm developed for [4]. The figure key summarizes the values of the system that will be used in the examples in this section.

Figure 3. List of Arrays



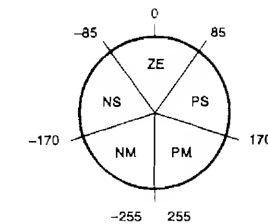
Fuzzy Logic Implementation as a Series of Loops

Key
nmembs=5
ninputs=2
nrules=11
noutputs=1
i, j, k = loops within loops

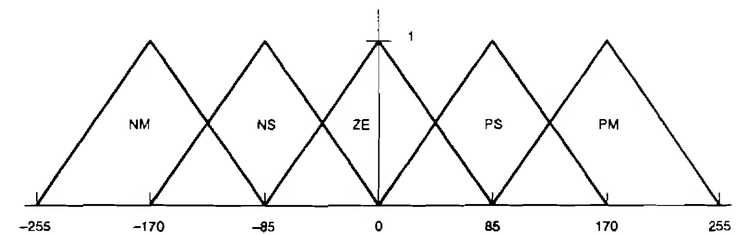
Note which arrays are modified in each section and their size boundaries. They are discussed later in more detail. The two inputs seen in x[ninputs] are position (found from the encoder) and velocity (found from an approximation of the derivative by taking the back-difference of the position). These are the Theta and dTheta variables described previously. As the compensator code begins, the position and velocity inputs (ErrNow and ErrDiff, respectively) are copied to the array X[ninputs]. The position is mapped so that one rotation of the motor ranges from –255 to +255 (See Figure 4 a). This relationship is mapped onto the x-axis of the membership function and thus fuzzifies the position of the motor (Figure 4 b). Note that this figure is not drawn to scale.

Figure 4. Motor Shaft/Membership Function Mapping

a. Motor Range

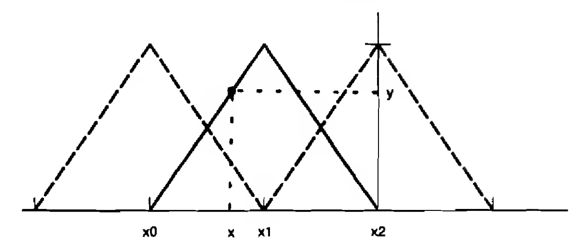


b. X-Axis Map



In the fuzzification loop, the degree of membership of each input relative to the input membership function is evaluated and written in the array X[ninputs][nmembs]. The value for a particular linguistic variable is the y value of the triangle for a particular value of x (See Figure 5).

Figure 5. Analysis for One Linguistic Variable



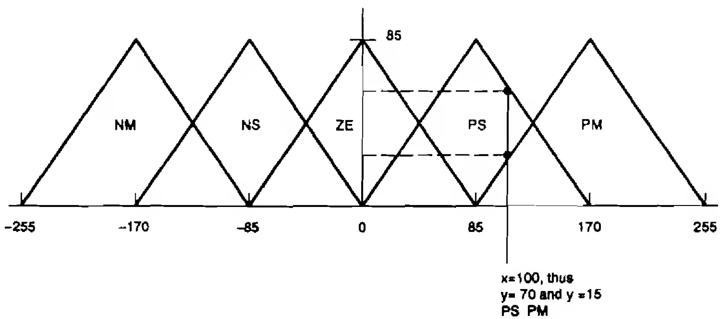
The y value is found by using the simple algebraic equation for a line ($y=mx+b$). This equation may be geometrically reduced to one of the two following equations, depending on which side of the triangle the value of x lies:

$$\begin{aligned} &if\ \{x : x0 < x < x1\}\ then\ y = (x - x0) \ / \ (x1 - x0)\ else \\ &if\ \{x : x1 < x < x2\}\ then\ y = (x2 - x) \ / \ (x2 - x1) \end{aligned} \tag{2}$$

The division needed is costly on most microprocessors, usually requiring at least a number of cycles equivalent to the number of bits of the number being divided. But if the slope m is made equal to 1 by causing the elements of the membership functions to be isosceles right triangles, the equation can be reduced to $y=x-x0$. This translates into a simple one-cycle subtraction. Note that forcing the elements of the membership functions to be isosceles right triangles also forces the peak of the membership function to no longer be 1. Rather the peak value=$x1-x0=x2-x1$=85 (as seen in Example 1). This action also eliminates the need for using a Q format [6] to represent the fractional values from Equation 2 if the triangle were not isosceles

Example 1 demonstrates this fuzzification calculation. If the position input value were 100 (i.e., x[0]=100), it would have nonzero degrees of membership in the PS and PM linguistic variables. You can also see this in Figure 5. To calculate the actual degree of membership value, the value for x is plugged into Equation 2 for both PS and PM boundaries. Thus, in PS the contribution is 70, while in PM it is 15. The rest of the linguistic variables are 0 because there is no contribution. (Of course, in software, all linguistic variables must be evaluated.) For this example, X[0][nmembs]=[0, 0, 0, 70, 15].

Example 1. Fuzzification Example



The next step involves fuzzy inference. In this loop, the maximum and minimum functions, as explained in [2], are implemented. In the actual code, the array indexing of the membership values is made nonnegative by adding a bias of three. Therefore, instead of NM to PM being indexed from −2 to +2, as seen in Figure 4, they are indexed from 0 to 4. Table 3 shows how the 11 rules are indexed and reindexed in the array RULE_TABLE[rule][input+output].

Table 3. Original and Reindexed Table of Rules

Original Index			Reindexed		
$x(0)$	$x(1)$	$u(0)$	$x(0)$	$x(1)$	$u(0)$
0	0	0	2	2	2
1	0	−1	3	2	1
2	0	−2	4	2	0
−1	0	1	1	2	3
−2	0	2	0	2	4
0	−1	1	2	1	3
0	−2	2	2	0	4
0	1	−1	2	3	1
0	1	−2	2	4	0
1	−1	0	3	1	2
−1	1	0	1	3	2

Some explanation is required for this decoding. The indexed rules match the explicit rules as described in Table 1. Also, the values given by accessing the RULE_TABLE are limited to the values indexed by nmembs. This characteristic is heavily used in the index manipulation and allows nmemb to be interchanged with RULE_TABLE[rule][input+output] in the appropriate parts of the algorithm.

To find the minimum value of X[ninputs][nmembs] decoded from the rule table inputs and stored in minZ (which is initialized to the maximum y value—in this case, 85), the equation is:

$$minZ = \min\ (X[input]\ [RULE\_TABLE\ [rule]\ [input]]) \tag{3}$$

(Since noutputs=1, the loop is simplified, and minZ does not need to be the general case array minZ[nrules]). Note that only the first two columns (the input columns) of RULE_TABLE are used in this part of the fuzzy inference section.

Then, for each rule, the max is taken of the output value U[nmembs], which is initialized to 0. The general case U[output][nmembs] is simplified because only one output is decoded from the rule table outputs and minZ. This equation can be summarized as:

$$\begin{aligned} U[nmembs] &= U(RULE\_TABLE[rule]\ [ninput + output]) \\ &= \max\ (U[RULE\_TABLE[rule]\ [ninput + output]],\ minZ) \end{aligned} \tag{4}$$

Note that in this section only the last column (the output column) is used. The following equation summarizes the minimum and maximum functions that are executed for each rule to result in the array U[nmembs] by plugging Equation 3 into Equation 4:

$$\begin{aligned} U[nmembs] = \max\ \{&U[RULE\_TABLE[rule]\ [ninput + output]]\ , \\ &\min\ (X[input]\ [RULE\_TABLE[rule]\ [input]])\} \end{aligned} \tag{5}$$

'he following example illustrates the fuzzy inference section. One cycle of the loop for rule=0 is shown or the input array:

$$X[ninputs]\ [nmembs] = \begin{vmatrix} 0 & 0 & 70 & 15 & 0 \end{vmatrix} \quad (6)$$
$$= \begin{vmatrix} 0 & 60 & 25 & 0 & 0 \end{vmatrix}$$

nd for rule=0, input=0, and input=1, plug into Equation 3:

$$minZ = min \begin{vmatrix} (X[0]\ [2]) \\ (X[1]\ [2]) \end{vmatrix} = min \begin{vmatrix} 70 \\ 25 \end{vmatrix} = 25 \quad (7)$$

'hen for the max rule=0, input=0, and input=1, plug into Equation 4:

$$U[nmembs] = U[2] = max \begin{vmatrix} U[2] \\ minZ \end{vmatrix} = max \begin{vmatrix} 0 \\ 25 \end{vmatrix} = 25 \quad (8)$$

'his process continues for the list of 11 rules so that the array U[nmembs] contains the maximum values of the min—i.e., the contribution of each rule to the inference.

'he final step involves defuzzifying the U[noutputs][nmembs] array. Since noutputs=1, U simplifies to U[nmembs]. The U[nmembs] array now has five values in it for its corresponding five positions. The center-of-gravity calculation is done with two loops. The first finds the numerator by using the multiplier to weight U[nmembs] by its position. The second loop finds the denominator by summing the position. The inevitable 16-bit divide loop then finds the output value u[output]. This u[output] represents the motor current mentioned in *Fuzzy Logic Theory for Servo Motor Control* (page 12). The divide operation is done on the 'C14 by a 16-cycle loop. This value is then scaled for the output and written to the NewServo memory location that sends it to the PWM generator.

'As an example of defuzzification,

f U[nmemb] = [0 15 70 35 0],

hen the output is:

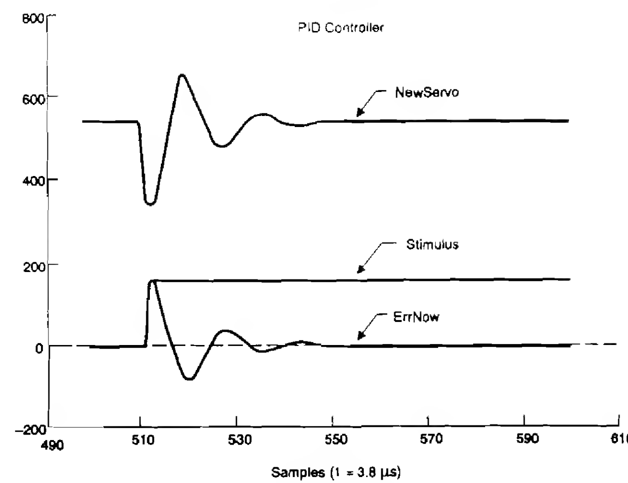
U[output] = 0*(–170) + 15*(–85) + 70*(0) + 35*(85) + 0*(170)
= 14.17

'hese loops thus evaluate the control input necessary for the servo motor.

Results

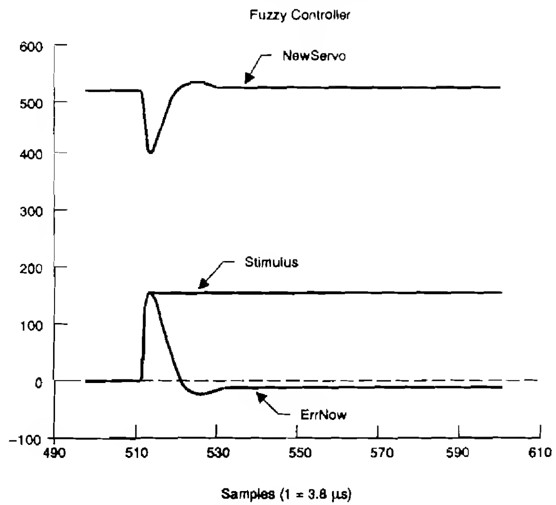
'he fuzzy logic code with two inputs, eleven rules, one output, and five linguistic variables requires about 2000 instruction cycles to execute on the TMS320C14. This means a 400-µs period (2.5-kHz frequency) because one instruction cycle is 200 nanoseconds on the 'C14. The update period of the motor is 3.8 ms (263-Hz frequency), so the fuzzy code is obviously adequate (for comparison, the PID code execution required nine µs (111-kHz frequency), for the update period). Figure 6 shows the PID performance for a step with the classical control overshoot and settling times.

Figure 6. PID Controller



The fuzzy logic curve (Figure 7) is smoother than the PID controller curve but doesn't go to zero.

Figure 7. Fuzzy Controller



The update period is adequate for applications such as servo motor, robotics, motion control, and automotive control. Better performance may be needed for such applications as hard-disk drives. Later-generation digital signal processors, such as the TMS320C5x, operate at up to 25 nanoseconds with much more efficient instruction code.

Conclusion

The final fuzzy logic system behaved favorably when compared to the conventional PID system. The system proved the feasibility of implementing a real-time fuzzy logic servo motor control. Proof of the fuzzy control was shown by positioning the motor spindle with an error outside the membership function, thus causing the control to desist. Further development could include a graphics display and the ability to vary the rules. The TMS320C14 board fits in a 12 x 8 x 6-inch suitcase conveniently and requires only an AC power supply and an RS-232 keyboard connection. This product easily demonstrates real-system fuzzy logic control on a microprocessor.

Fuzzy logic has great potential as a programmable solution for general engineering. For applications where performance is a priority, a hard-wired silicon solution (which may even configure as a microprocessor peripheral) based on [4] is being developed. The programmable solution may assist in the transition to this hard-wired option, depending on the software/hardware tradeoffs.

APPENDIX A
PID Code

```
.title "Servo compensator            $Revision:  3.4  $"
;***************************************************************************
; $Header::  C:/src/c14/ps/vcs/comp.s_v   3.4   01 Oct 1991 17:24:18   "$
;
; $config$="/T8 /K! /L;*.ref.def /R:*– /B80"
; !config!="/Mcomp.s"
;
; !NAME!
;             comp.s
;
; !PATHS!
;             modules
;                           !0!
; DESCRIPTION
;             Servo compensator – uses a PID algorithm
;
; PRINCIPLE AUTHORS:
;             Dave Sewhuk
;
; CREATION DATE:
;             December 23, 1990  22:54:08
;
; COPYRIGHT NOTICE:
;             (C)Copyright 1990 Teknic Inc.  All rights reserved.
;
; !end!
;***************************************************************************

;***************************************************************************
; HEADERS UTILIZED
             .include "macrodef.inc"
             .include "c14io.inc"
;
;***************************************************************************

;***************************************************************************
; !NAME!
;             comp.s
;
; !PATHS!
;             Exported Variables
; !0!
             .def   PwmChannel,PwmPeriod
             .def   NewServo,ErrDiff,Kintegrator
             .def   ErrNow,ErrLast,DesiredPosition
```

```
    SUB     PwmPeriod,1          ; Make largest negative
    SACL    NewServo
;
; Set Output PWM to new value
;
pwmisrSet:
    LAC     PwmPeriod,1          ; Get center period value
    ADD     NewServo             ; Add newly computed value
    SACL    NewServo
    .if     ChipV1R1             ; PWM bug workaround
    .ref    CONST3
    SUB     CONST3
    BLEZ    pwmisr10             ; No fixes needed
    LAC     NewServo             ; Lower 2 bits set?
    AND     CONST3
    SUB     CONST3
    BNZ     pwmisr10             ; OK, No fixes needed
    LACK    4
    ADD     NewServo
    SACL    NewServo
pwmisr10:
    .endif
    LACK    ActionBank           ; Set bank for timers
    SACL    ISR_TMP
    OUT     ISR_TMP,BSR
    OUT     NewServo,ACT0        ; Output channel 1 timer
    LAC     PwmPeriod,2          ; Calculate complimentary output
    SUB     ONE,2
    SUB     NewServo
    SACL    NewServo
    OUT     NewServo,ACT1        ; Set complimentary channel 2 timer
    RET                          ; All done.. back to the ISR already
                                 ; in progress.
; !skip end!
; !END!

;=====================================================================
===
; END OF FILE
;=====================================================================
===
    .end
```


References

[1] B. Kosko. *Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1992.

[2] Y.F. Li and C.C. Lau. "Development of Fuzzy Algorithms for Servo Systems," *IEEE Control Systems Magazine.* April, 1989, pp. 65-71.

[3] K. Self. "Designing with Fuzzy Logic," *IEEE Spectrum.* November, 1990, pp. 42-44, 105.

[4] P. Thrift. *The Programmable Fuzzy Logic Array.* Dallas, Texas: Texas Instruments Central Research Laboratories, 1992.

[5] *Power-14/Power Source User's Manual.* Rochester, New York: Teknic, Inc., 1989.

[6] *TMS320C1x User's Guide.* Dallas, Texas: Texas Instruments, 1991.

The Programmable Fuzzy Logic Array

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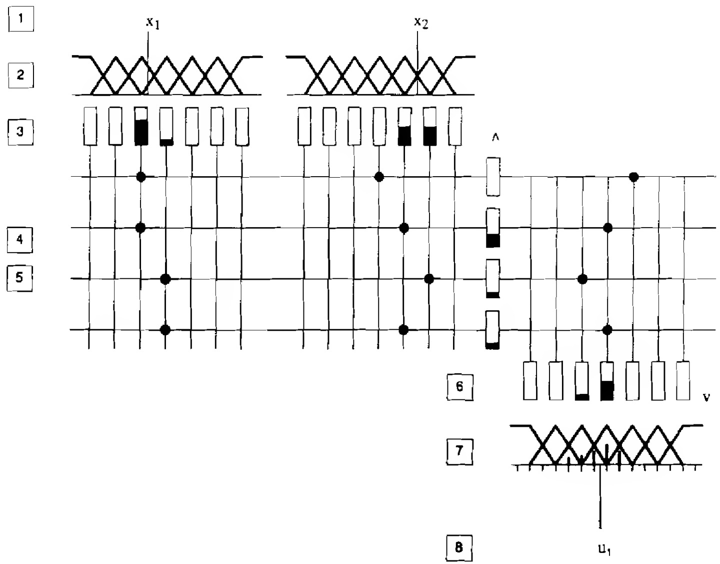
Introduction

The Programmable Fuzzy Logic Array (PFLA) produces nonlinear multidimensional mappings by encoding fuzzy rule systems in a programmable array architecture. It can be used as a component of a graphical user interface (GUI) for designing fuzzy controllers, as well as a software blueprint for mapping onto VLSI hardware. An advantage of the PFLA over other fuzzy representations, such as the FAM (Fuzzy Associative Memory) [2], is the PFLA's ability to easily visualize several inputs and outputs simultaneously.

Data Flow

Figure 1 shows the PFLA data flow. Succeeding text describes PFLA in general mapping terms from P inputs to Q outputs, but only $P = 2$ and $Q = 1$ are shown in Figure 1. Each item number in the text corresponds to a step of Figure 1.

Figure 1. PFLA Data Flow



1. Each input $x_i : i = 1,\ldots, P$ to the PFLA is a numerical value in a range $[a_i,b_i]$.
2. Defined over each input range $[a_i, b_i] : i = 1,\ldots, P$ are fuzzy membership functions $F^1_i,\ldots, F^{n_i}_i$, where n_i is the number of membership functions defined for input i. Each membership function varies between 0 and 1. The cases shown in Figure 1 are trapezoidal membership functions, which are defined in Appendix A. Other parametric families of membership functions can be substituted. Also, for each fuzzy membership function F^j_i, there is a corresponding label. A typical labeling scheme (labels are not shown in Figure 1) for $n_i = 7$ is: negative large (NL), negative medium (NM), negative small (NS), zero (ZE), positive small (PS), positive medium (PM), positive large (PL). Although the fuzzy sets for each input in Figure 1 appear symmetrically spaced, these are not necessarily the optimal settings. If trapezoidal memberships functions are used, four numbers $t^j_i = [t^j_{i,1}, t^j_{i,2}, t^j_{i,3}, t^j_{i,4}]$ must be specified for each membership function F^j_i.
3. Each input x_i is evaluated by each of its fuzzy membership functions to produce a value f^j_i:

$$f^j_i = F^j_i(x_i) : i = 1,\ldots, P \; j = 1,\ldots, n_i \tag{9}$$

These values appear in the boxes as shown as a thermometer level. f^j_i also refers to the box that contains its value. In the example shown, only two boxes for each input have positive evaluation.

4. Fuzzy rules are encoded in a crossbar pattern. Corresponding to each of the input boxes f^j_i in 3, above, is a vertical wire dropping down. A horizontal wire crosses those vertical wires and also the vertical wires corresponding to output boxes in 6, below. A connection is indicated by a •. For each horizontal wire, there is, at most, one connection per input and output variable. Each horizontal connection pattern encodes a rule. For example, the connection pattern on the first horizontal line encodes the rule:

$$\textit{If } x_1 \textit{ is NS} \text{ and } x_2 \textit{ is ZE, then } u_1 \textit{ is PS.} \tag{10}$$

Four rules are shown in Figure 1. R rules can be specified by a table of numbers:

$$r^1_1, \ldots, r^P_1, s^1_1, \ldots s^Q_1 \tag{11}$$

$$r^1_R, \ldots, r^P_R, s^1_R, \ldots s^Q_R \tag{12}$$

where r^i_k is in $\{1,\ldots,n_i,\text{NULL}\}$, s^j_k is in $\{1,\ldots,m_j,\text{NULL}\}$. Here, m_j is the number of fuzzy sets for output j. The kth horizontal wire specifies the rule

$$F^{r^1_k}_1,\ldots, F^{r^P_k}_P \rightarrow G^{s^1_k}_1,\ldots, G^{s^Q_k}_Q \tag{13}$$

The NULL indicates that there is no connection for this input/output variable. In Figure 1, the connections would be

3,4,5
3,5,4
4,6,3
4,5,4

5. **Intercepting the horizontal wires between the inputs and outputs are the $\wedge$ ("wedge") boxes. A conventional $\wedge$ operator is the numerical minimum of the values (this is used in Figure 1), but other operators are possible (for example, product, or any of a set of so-called t-norms [1]).**

At the kth $\wedge$ box, this is produced:

$$h_k = \wedge (f_1^{r_k^1}, f_2^{r_k^2}, \ldots, f_P^{r_k^P}) \quad (14)$$

If r_k^i is NULL, this argument is omitted.

6. The $\vee$ ("vee") boxes are computed according to the values of the connections above them. A conventional $\vee$ operator is the numerical maximum of the values (see Figure 1). The values of the $\vee$ boxes are

$$g_i^j = \vee [h_k : s_k^i = j] \; i = 1,\ldots,Q \;\; j = 1,\ldots,m_i \quad (15)$$

Other operators [1] can be substituted for this operator (for example, probabilistic sum: $x \oplus y = x + y - xy$, etc.).

7. Defined over each output range $[c_i,d_i] : i = 1,\ldots,Q$ are fuzzy membership functions $G_i^1,\ldots,G_i^{m_i}$, where m_i is the number of membership functions defined for output i. Also defined for each output i is a vector l_i of locations quantizing the range: $1_i = [l_i^1,\ldots l_i^{q_i}]$, where q_i is the number of locations specified for output i. In Figure 1, 17 locations are shown for output 1.

$$w_i^j = G_i^j(1_i), \; i = 1,\ldots,Q \;\; j = 1,\ldots, m_i \quad (16)$$

Here, G_i^j is applied by component to get the resulting vector. The w_i^j are precomputed and stored for each $\vee$ box. Then, this is computed for output i:

$$v_i = \vee^* (w_i^1 \wedge^* g_i^1, \ldots, w_i^{m_i} \wedge^* g_i^1) \quad (17)$$

Here, the "wedge" and "vee" operators are not necessarily the ones used in the boxes above. In Figure 1, v_1 is shown as a sequence of vertical bars; $\vee^*$ is the maximum operation, and $\wedge^*$ is the product.

8. The final stage is to compute u_i from v_i for each output:

$$u_i = \frac{v_i \cdot 1_i}{v_i \cdot 1} \quad (18)$$

where $1 = [1,\ldots,1]$ is a vector of 1s.

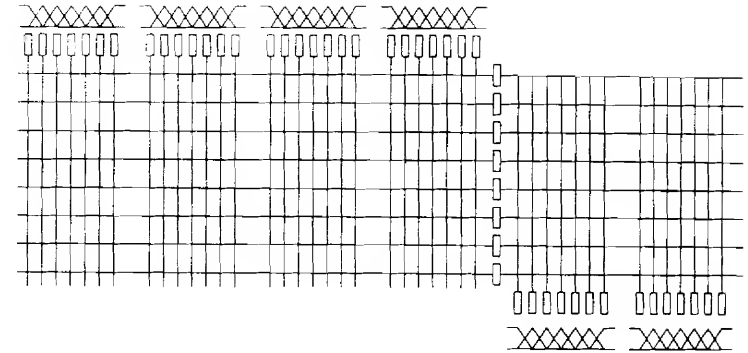
If, corresponding to each output fuzzy set G_i^j, there is a single distinct location l_i^j, $w_i^j = G_i^j(l_i^j)$, and $\wedge^*$ is the product operation, then the above defuzzification procedure reduces to

$$u_i = \frac{w_i^1 \cdot g_i^1 \cdot l_i^1 + \ldots + w_i^{m_i} \cdot g_i^{m_i} \cdot l_i^{m_i}}{w_i^1 \cdot g_i^1 + \ldots + w_i^{m_i} \cdot g_i^{m_i}} \quad (19)$$

This provides "weighted point mass defuzzification". In this case, only a "weight" w_i^j and a location l_i^j must be stored for each $\vee$ box.

Figure 2 shows the state of the system for particular inputs x_1,x_2. Thermometer levels indicate the values in both the fuzzification and the $\wedge$ and $\vee$ boxes (here, *min* and *max* are the operations performed). Darkened rule connections indicate values that propagated through the array (since *min* and *max* are used, on each output line there is, in general, one *max* winner and one *min* winner). Figure 2 also shows the layout for a four-input, two-output (4-2) system. In a GUI, you can use point-and-clicks to set the rule connections and membership function positioning.

Figure 2. PFLA 4–2 Layout



Summary

This is the information that provides the setting for the PFLA:

#Inputs: P	#Input fuzzy sets: $n_1,\ldots,n_P$
#Outputs: Q	#Output fuzzy sets: $m_1,\ldots,m_Q$
Input fuzzy sets	$[a_i;b_i]$, $f_i^1,\ldots,f_i^{n_i}$, $i = 1,\ldots, P$
#Rules: R	Operators: $\wedge \; \vee$
#Rule table:	$r_k^1,\ldots,r_k^P, s_k^1,\ldots s_k^Q$, $k = 1,\ldots, R$
Output defuzzifiers	$[c_j,d_j], l_j, w_j^1,\ldots,w_j^{m_j}$, $j = 1,\ldots, Q$ $\;\wedge^* \; \vee^*$

Appendix A

A trapezoidal membership function on an interval [a,b] is specified by four numbers t_1, t_2, t_3, t_4, satisfying

$$a \le t_1 \le t_2 \le t_3 \le t_4 \le b$$

The trapezoidal function is defined by

$$TZ(a,t_1,t_2,t_3,t_4,b)(x) \begin{cases} 0 & \text{for x in } [a,t_1] \\ (x-t_1)/(t_2-t_1) & \text{for x in } (t_1,t_2) \\ 1 & \text{for x in } [t_2,t_3] \\ (t_4-x)/(t_4-t_3) & \text{for x in } (t_3,t_4) \\ 0 & \text{for x in } [t_4,b \end{cases}$$

Note that if $t_1 = t_2$ *or* $t_3 = t_4$, this part of the definition is void.

References

[1] Pedrycz, W. *Fuzzy Control and Fuzzy Systems.* John Wiley & Sons Inc., 1989.

[2] Kosko, B. *Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence.* Prentice-Hall, Inc., 1992.